Some More Results on Power 3 Mean Graphs

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Abstract:

A graph G with p vertices and q edges is called a power -3 mean graph,if it is possible to label the vertices $v \in V$ with distinct labels $f(x)$ from $1,2,\dots\dots,q+1$ in such a way that in each edge $e=uv$ is labeled with $f(e=uv) = \left\lceil \left(\frac{x^3+y^3}{2}\right)^{\frac{1}{3}} \right\rceil$ or $\left\lfloor \left(\frac{x^3+y^3}{2}\right)^{\frac{1}{3}} \right\rfloor$. Then , the edge labels are distinct. In this case f is called Power 3 Mean labeling of G.In this case, f is a Power 3 mean labeling of G and G is called a Power 3 Mean Graph**.**

Key words:

Graph, Power 3 Mean Graphs, Path,Cycle,Complete graph,Complete bipartite graphs.

Introduction:

The graph considered here will be finite,undirected and simple.The vertex set is denoted by $V(G)$ and the edge set is denoted by $E(G)$.For all detailed survey of graph labeling we refer Gallian[1].For all other standard terminology and notations we follow Harary[2].S.S.Sandhya and S.Sreeji introduced the concept of Power 3 Mean labelling of graphs.The definition and other informations which are useful for the present investigation are given below.

Definition:1.1: A walk in which $u_1,u_2,u_3,\dots\dots u_n$ is distinct is called a path.A path on n vertices is denoted by P_n.

Definition:1.2: A closed path is called a cycle. A cycle on n vertices is denoted by C_n.

Definition:1.3: The corona of two graphs G_1 and G_2 is the graph $G = G_1 \odot G_2$ formed by one copy of G_1 and $|G_1|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2.

Definition:1.4: A graph G is said to be complete if every pair of its distinct vertices are adjacent.A complete graph on n vertices is denoted by K_n.

Definition:1.5: An (n,t)- kite graph consists of a cycle of length n with t edges path attached to one vertex of a cycle.

Definition:1.6: A complete bipartite graph is a complete graph with bipartition (V_1, V_2) such that every vertex of V_1 is joined to all the vertices of V_2.It is denoted by $K_{m,n}$ where $|V_1| = m$ and $|V_2| = n$.

Theorem:1.7: Any path is a Power 3 Mean Graph.

Theorem:1.8: Any cycle is a Power 3 Mean Graph.

Theorem:1.9: Combs are Power 3 Mean Graph.

Theorem:1.10: Complete graph is a Power 3 mean graph iff $n < 3$.

Theorem:1.11: $K_{1,n}$ is a Power 3 mean graph iff $n \leq 8$

2.Main Results:

Theorem:2.1

Let P_n be the path and G be the graph obtained from P_n by attaching C_3 in both the ends of P_n.Then G is a Power 3 Mean Graph.

Proof:

Let P_n be a path $u_1, u_2, u_3, \ldots \ldots u_n$ and $v_1u_1u_2, v_2u_{n-1}u_n$ be the triangles which are connected to the path at the end.

Define a function

$f: V(G) \rightarrow 1,2, \ldots .. q + 1$ by $f(u_i) = i + 1,\ 1 \leq i \leq n-1,$

$$f(u_n) = n + 3$$

$$f(v_1) = 1$$

$$f(v_2) = n + 2$$

Then the edges are labeled as,

$$f(u_1v_1) = 1$$

$$f(u_{n-1}v_2) = n + 1$$

$$f(u_nv_2) = n + 3$$

$$f(u_{n-1}u_n) = n + 2$$

$$f(u_iu_{i+1}) = i + 2\ ;\ 1 \leq i \leq n - 1$$

Hence f is a Power 3 Mean labeling.

Example 2.2:

A Power 3 mean labeling of G obtained from P_7 is given below.

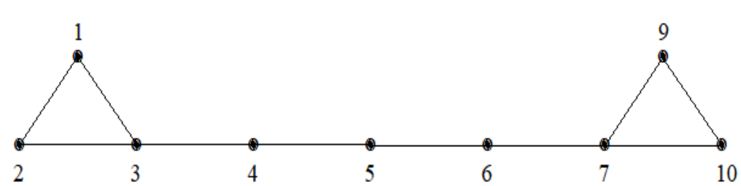

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Figure : 1

Theorem: 2.3:

Let G be the graph obtained by attaching a pendant edge to both the sides of each vertex of a path P_n.Then G is a Power 3 Mean Graph.

Proof:

Let G be a graph obtained by attaching pendant edges to both the sides of each ertex of a path P_n .

Let u_i, v_i and w_i ; $1 \le i \le n$ be the new vertices of G.

Define a function

$$f: V\ (G) \to \{1,2,3, \dots\ , q\ +\ 1\} \text{ by}$$

$$f(u_i) = 3i - 2\ ; 1 \le i \le n$$

$$f(v_i) = 3i\ ; 1 \le i \le n$$

$$f(w_i) = 3i - 1\ ; 1 \le i \le n$$

The edge labels are,

$$f(u_i u_{i+1}) = 3i\ ;\ 1 \le i \le n$$

$$f(u_i v_i) = 3i - 1\ ;\ 1 \le i \le n$$

$$f(u_i w_i) = 3i - 2\ ;\ 1 \le i \le n$$

Hence f is a Power 3 mean labeling.

Example 2.4:

The graph obtained from P_6 is given below.

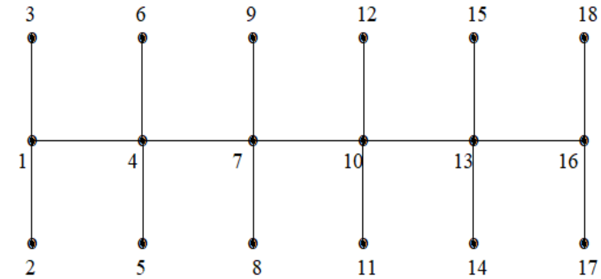

Figure : 2

Theorem: 2.5:

Let G be the graph obtained by attaching each vertex of P_n to the central vertex of $K_{1,2}$.Then G is a Power 3 Mean Graph.

Proof:

Let P_n be the path $u_1 u_2 \dots \dots . u_n$ and let v_i, w_i be the vertices of $K_{1,2}$ which are attached to the vertex u_i of P_n.

Define a function

$$f: V\ (G) \to \{1,2,3, \dots\ , q\ +\ 1\} \text{ by}$$

$$f(u_i) = 3i - 2\ ; 1 \le i \le n$$

$$f(v_i) = 3i - 1\ ; 1 \le i \le n$$

$$f(w_i) = 3i\ ; 1 \le i \le n$$

The edge labels are,

$$f(u_i u_{i+1}) = 3i\ ;\ 1 \le i \le n$$

$$f(u_i v_i) = 3i - 2\ ;\ 1 \le i \le n$$

$$f(u_i w_i) = 3i - 1\ ;\ 1 \le i \le n$$

Hence , f is a Power 3 mean labeling.

Example 2.6:

Power 3 mean labeling of G obtained from P_4 is given below.

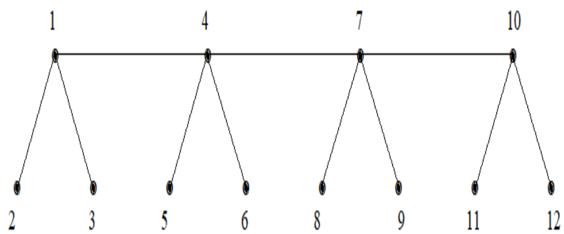

Figure : 3

Theorem: 2.7:

Let G be the graph obtained by attaching each vertex of P_n to the central vertex of $K_{1,3}$.Then G is a Power 3 Mean Graph.

Proof:

Let P_n be the path $u_1u_2 \dots \dots . u_n$ and let v_i, w_i be the vertices of $K_{1,3}$ which are attached to the vertex u_i of P_n.

Define a function

$$f: V\ (G) \to \{1,2,3, \dots\ , q\ +\ 1\} \text{ by}$$

$$f(u_i) = 4i - 3\ ; 1 \le i \le n$$

$$f(v_i) = 4i - 2\ ; 1 \le i \le n$$

$$f(w_i) = 4i - 1\ ; 1 \le i \le n$$

$$f(t_i) = 4i\ ; 1 \le i \le n$$

The edge labels are,

$$f(u_iu_{i+1}) = 4i\ ;\ 1 \le i \le n$$

$$f(u_iv_i) = 4i - 3\ ;\ 1 \le i \le n$$

$$f(u_iw_i) = 4i - 2\ ;\ 1 \le i \le n$$

$$f(u_it_i) = 4i - 1\ ;\ 1 \le i \le n$$

Hence f is a Power 3 mean labeling.

Example 2.8:

Power 3 mean labelling of G obtained from $P_4 \odot K_{1,3}$ is given below.

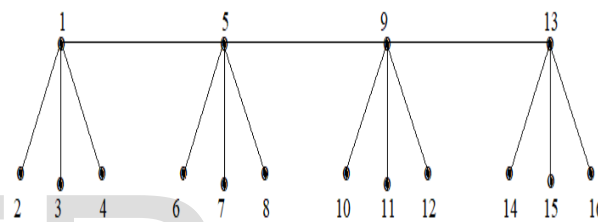

Figure : 4

Theorem: 2.9:

A (m, n)- Kite graph is a Power 3 mean graph.

Proof:

Let $u_1u_2 \dots \dots . u_m\, u_1$be the given cycle of length m $v_1v_2 \dots \dots v_n$ be the given path of length n.

Define a function

$$f: V\ (G) \to \{1,2,3, \dots\ , q\ +\ 1\} \text{ by}$$

$$f(u_i) = i\ ; 1 \le i \le m$$

$$f(v_i) = m + i\ \ ; 1 \le i \le n$$

Then the edge labels are distinct.

Hence (m, n)- Kite graph is a Power 3 mean graph.

Example 2.10:

The Power 3 mean labelling of (5,6)-Kite graph is given below.

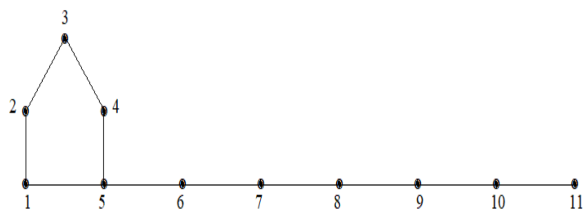

Figure : 5

Theorem: 2.11:

Let G be the graph obtained by joining a vertex of degree two of a comb.Then G is a Power 3 Mean Graph.

Proof:

Let the vertex set of a comb be $\{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$.Let P_n be the path $u_1 u_2 \dots \dots u_n$ .Join a vertex v_i to u_i , $1 \leq i \leq n$.Let G be a graph obtained by joining a pendent vertex w to u_n.

Define a function

$$f: V(G) \to \{1,2,3, \dots, q+1\} \text{ by}$$

$$f(u_i) = 2i - 1 \,; 1 \leq i \leq n$$

$$f(v_i) = 2i \,; 1 \leq i \leq n$$

$$f(w) = 2n + 1 \;\; ; 1 \leq i \leq n$$

Then the edges are labelled as,

$$f(u_i u_{i+1}) = 2i \,;\; 1 \leq i \leq n - 1$$

$$f(u_i v_i) = 2i - 1 \,;\; 1 \leq i \leq n$$

$$f(u_n w) = 2n.$$

Hence f is a Power 3 mean labelling of G.

Example 2.12:

Power 3 mean labelling of G with 11 vertices and 10 edges is given below.

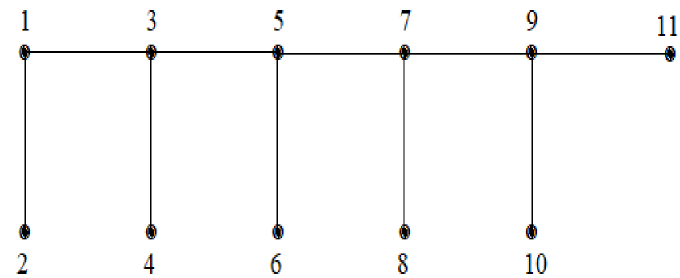

Figure : 6

In the similar manner , we can see the Power 3 Mean Labeling of G obtained by joining a pendant vertex with a vertex of degree two on both ends of a comb graph.Power 3 Mean labeling of G with 10 vertices and 9 edges is shown below.

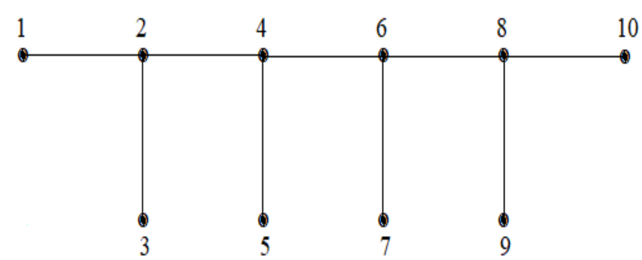

Figure : 7

Conclusion:

All graphs are not Power 3 mean graphs.It is very interesting to investigate graphs which admits Power 3 Mean labelling. In this paper we proved that some graphs are Power 3 Mean graphs. It is possible to investigate similar results for several other graphs.

Aknowledgement:

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